Rotatory Thermosolutal Convection in a Couple-Stress Fluid

Pardeep Kumar and Mahinder Singh

Department of Mathematics, ICDEOL, Himachal Pradesh University, Shimla-171005, India

Reprint requests to Dr. P. K.; E-mail: drpardeep@sancharnet.in

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The thermosolutal instability of couple-stress fluid in the presence of uniform vertical rotation is considered. Following the linearized stability theory and normal mode analysis, the dispersion is obtained. For the case of stationary convection, the stable solute gradient and rotation have stabilizing effects on the system, whereas the couple-stress has both stabilizing and destabilizing effects. The dispersion relation is also analyzed numerically. The stable solute gradient and the rotation introduce oscillatory modes in the system, which did not occur in their absence. The sufficient conditions for the non-existence of overstability are also obtained.

Key words: Thermosolutal Convection; Couple-Stress Fluid; Uniform Vertical Rotation.

1. Introduction

The theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydrodynamics has been treated in detail by Chandrasekhar [1] in his celebrated monograph. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [2]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The problem of the onset of thermal instability in the presence of a solute gradient is of great importance because of its applications to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The thermosolutal convection problems also arise in oceanography, limnology and engineering. Stomell et al. [3] did the pioneering work regarding the investigation of thermosolutal convection. This work was elaborated in different physical situations by Stern [4] and Nield [5]. A double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose, has been explained by Brakke [6]. Nason et al. [7] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultra centrifuge.

The theory of couple-stress fluid has been formulated by Stokes [8]. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrications of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. The shoulder, ankle, knee, and hip joints are the loaded-bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is generally clear or yellowish. According to the theory of Stokes [8], couple-stresses appear in noticeable magnitudes in fluids with very large molecules.

Many of the flow problems in fluids with couple-stresses, discussed by Stokes, indicate some possible experiments, which could be used for determining the material constants, and the results are found to differ from those of Newtonian fluid. Couple-stresses are found to appear in noticeable magnitudes in polymer solutions for force and couple-stresses. This theory is developed in an effort to examine the simplest generalization of the classical theory, which would allow polar effects. The constitutive equations proposed by Stokes [8] are:

$$\mathbf{T}_{(ij)} = (-p + \lambda \mathbf{D}_{kk})\delta_{ij} + 2\mu \mathbf{D}_{ij},$$

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$$\mathbf{T}_{[ij]} = -2\eta \mathbf{W}_{ij.kk} - \frac{\rho}{2}\boldsymbol{\varepsilon}_{ijs} G_s,$$

and

$$\mathbf{M}_{ij} = 4\eta \boldsymbol{\omega}_{j,i} + 4\eta' \boldsymbol{\omega}_{i,j},$$

where

$$\mathbf{D}_{ij} = \frac{1}{2}(V_{i,j} + V_{j,i}), \quad \mathbf{W}_{ij} = -\frac{1}{2}(V_{i,j} - V_{j,i})$$

and $\boldsymbol{\omega}_i = \frac{1}{2}\boldsymbol{\varepsilon}_{ijk} V_{k,j}$.

Here $\mathbf{T}_{ij}$, $\mathbf{T}_{(ij)}$, $\mathbf{T}_{[ij]}$, $\mathbf{M}_{ij}$, $\mathbf{D}_{ij}$, $\boldsymbol{W}_{i,j}$, $\boldsymbol{\omega}_i$, G_s, $\boldsymbol{\varepsilon}_{ijk}$, V, ρ, and λ, μ, η, η', are stress tensor, symmetric part of $\mathbf{T}_{ij}$, anti-symmetric part of $\mathbf{T}_{ij}$, the couple-stress tensor, deformation tensor, the vorticity tensor, the vorticity vector, body couple, the alternating unit tensor, velocity field, the density, and material constants, respectively. The dimensions of λ and μ are those of viscosity whereas the dimensions of η and η' are those of momentum.

Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [9] modeled the synovial fluid as a couple-stress fluid. The synovial fluid is the natural lubricant of joints of the vertebrates. The detailed description of the joint lubrication has very important practical implications. Practically all diseases of joints are caused by or connected with a malfunction of the lubrication. The efficiency of the physiological joint lubrication is caused by several mechanisms. The synovial fluid is, due to its content of the hyaluronic acid, a fluid of high viscosity, near to a gel. Goel et al. [10] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Sharma et al. [11] have considered a couple-stress fluid with suspended particles heated from below. They have found that for stationary convection, couple-stress has a stabilizing effect whereas suspended particles have a destabilizing effect. In another study, Sunil et al. [12] have considered a couple stress fluid heated from below in a porous medium in the presence of a magnetic field and rotation. Kumar et al. [13] have considered the thermal instability of a layer of a couple-stress fluid acted on by a uniform rotation, and have found that for stationary convection, the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects.

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics and various applications mentioned above, the thermosolutal convection in couple-stress fluid in the presence of uniform vertical rotation has been considered in the present paper.

2. Formulation of the Problem and Perturbation Equations

Here we consider an infinite, horizontal incompressible couple-stress fluid layer of thickness d, heated and soluted from below so that the temperatures, densities and solute concentrations at the bottom surface $z = 0$ are T_0, ρ_0 and C_0, and at the upper surface $z = d$ are T_d, ρ_d and C_d, respectively, and that a uniform temperature gradient $\beta = |\mathrm{d}T/\mathrm{d}z|$ and a uniform solute gradient $\beta' = |\mathrm{d}C/\mathrm{d}z|$ are maintained. The gravity field $\boldsymbol{g}(0,0,-g)$ and a uniform vertical rotation $\boldsymbol{\Omega}(0,0,\Omega)$ act on the system.

Let $\mathbf{T}_{ij}$, $\boldsymbol{\tau}_{ij}$, $\boldsymbol{e}_{ij}$, $\boldsymbol{\delta}_{ij}$, μ, μ', v_i and x_i denote the stress tensor, shear stress tensor, rate-of-strain tensor, Kronecker delta, viscosity, couple-stress viscosity, velocity vector and position vector, respectively. The constitutive relations for the couple-stress fluids are

$$\begin{aligned}
\mathbf{T}_{ij} &= -p\delta_{ij} + \tau_{ij}, \\
\boldsymbol{\tau}_{ij} &= 2(\mu - \mu' \nabla^2) e_{ij}, \\
\boldsymbol{e}_{ij} &= \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right).
\end{aligned}$$

Let p, ρ, T, C, α, α', $\boldsymbol{g}(0,0,-g)$ and $\boldsymbol{q}(u,v,w)$ denote, respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of couple-stress fluid [1, 2, 8] are

$$\frac{\partial \boldsymbol{q}}{\partial t} + (\boldsymbol{q}\cdot\nabla)\boldsymbol{q} = -\frac{1}{\rho_0}\nabla p + \boldsymbol{g}\left(1 + \frac{\delta\rho}{\rho_0}\right) + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\boldsymbol{q} + 2(\boldsymbol{q}\times\boldsymbol{\Omega}), \tag{1}$$

$$\nabla\cdot\boldsymbol{q} = 0, \tag{2}$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{q}\cdot\nabla)T = \chi\nabla^2 T, \tag{3}$$

$$\frac{\partial C}{\partial t} + (\boldsymbol{q}\cdot\nabla)C = \chi'\nabla^2 C, \tag{4}$$

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)], \tag{5}$$

where the suffix zero refers to the values at the reference level $z = 0$ and in writing (1) use has been made of Boussinesq approximation. The viscosity μ, couple-stress viscosity μ', kinematic viscosity ν, thermal diffusivity χ and the analogous solute diffusivity χ' are all assumed to be constants. The steady state solution is

$$\begin{aligned} &\boldsymbol{q} = (0,0,0), \quad T = T_0 - \beta z, \quad C = C_0 - \beta' z, \\ &\rho = \rho_0(1 + \alpha\beta z - \alpha'\beta' z'), \end{aligned} \tag{6}$$

where $\beta = (T_0 - T_1)/d$ and $\beta' = (C_0 - C_1)/d$ are the magnitudes of uniform temperature and concentration gradients and are both positive as temperature and concentration decrease upwards.

Let δp, $\delta\rho$, θ, γ and $\boldsymbol{q}(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C and velocity $\boldsymbol{q}(0,0,0)$. The change in density $\delta\rho$, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \tag{7}$$

Then the linearized hydrodynamic perturbation equations are

$$\begin{aligned} \frac{\partial \boldsymbol{q}}{\partial t} &= -\frac{1}{\rho_0}\nabla\delta p - \boldsymbol{g}(\alpha\theta - \alpha'\gamma) \\ &\quad + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\boldsymbol{q} + 2(\boldsymbol{q}\times\boldsymbol{\Omega}), \end{aligned} \tag{8}$$

$$\nabla\cdot\boldsymbol{q} = 0, \tag{9}$$

$$\frac{\partial\theta}{\partial t} = \beta w + \chi\nabla^2\theta, \tag{10}$$

$$\frac{\partial\gamma}{\partial t} = \beta' w + \chi'\nabla^2\gamma. \tag{11}$$

Within the framework of the Boussinesq approximation, (8) – (11) give

$$\begin{aligned} &\frac{\partial}{\partial t}\nabla^2 w - g\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\alpha\theta - \alpha'\gamma) + 2\Omega\frac{\partial\zeta}{\partial z} \\ &= \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^4 w, \end{aligned} \tag{12}$$

$$\frac{\partial\zeta}{\partial t} - 2\Omega\frac{\partial w}{\partial z} = \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\zeta, \tag{13}$$

$$\left(\frac{\partial}{\partial t} - \chi\nabla^2\right)\theta = \beta w, \tag{14}$$

$$\left(\frac{\partial}{\partial t} - \chi'\nabla^2\right)\gamma = \beta' w, \tag{15}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ denotes the z-component of the vorticity.

3. Dispersion Relation

We now analyze the disturbances into normal modes, assuming that the perturbation quantities are of the form

$$\begin{aligned} &[w,\theta,\gamma,\zeta] = \\ &[W(z),\Theta(z),\Gamma(z),Z(z)]\exp(ik_x x + ik_y y + nt), \end{aligned} \tag{16}$$

where k_x, k_y are the wave numbers along x- and y-directions, respectively, $k = (\sqrt{k_x^2 + k_y^2})$ is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using expression (16), (12) – (15) in non-dimensional form become

$$\begin{aligned} &\left[\sigma(D^2 - a^2)W + \frac{ga^2d^2}{\nu}(\alpha\Theta - \alpha'\Gamma) + \frac{2\Omega d^3}{\nu}DZ\right] = \\ &[1 - F(D^2 - a^2)](D^2 - a^2)^2 W, \end{aligned} \tag{17}$$

$$[\sigma - \{1 - F(D^2 - a^2)\}(D^2 - a^2)]Z = \frac{2\Omega d}{\upsilon}DW, \tag{18}$$

$$(D^2 - a^2 - p_1\sigma)\Theta = -\left(\frac{\beta d^2}{\chi}\right)W, \tag{19}$$

$$(D^2 - a^2 - q\sigma)\Gamma = -\left(\frac{\beta' d^2}{\chi'}\right)W, \tag{20}$$

where we have put $a = kd$, $\sigma = \frac{nd^2}{\nu}$, $\frac{x}{d} = x^*$, $\frac{y}{d} = y^*$, $\frac{z}{d} = z^*$, and $D = \frac{d}{\mathrm{d}z^*}$. Here $p_1 = \frac{\nu}{\chi}$ is the Prandtl number, $q = \frac{\nu}{\chi'}$ is the Schmidt number, and $F = \frac{\mu'}{\rho_0 d^2\nu}$ is a dimensionless couple-stress parameter.

We consider the case where both boundaries are free as well as perfect conductors of both heat and solute concentrations. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which (17) – (20) must be solved, are

$$\begin{aligned} &W = D^2W = D^4W = 0, \quad \Theta = 0, \quad \Gamma = 0, \\ &DZ = 0, \ \text{at} \ z^* = 0 \ \text{and} \ 1. \end{aligned} \tag{21}$$

The case of two free boundaries, though little artificial, is the most appropriate for stellar atmospheres [14]. Dropping the stars for convenience and using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish on the boundaries and hence, the proper solution of W charactering the lowest mode is

$$W = W_0 \sin \pi z, \tag{22}$$

where W_0 is a constant.

Eliminating Θ, Γ and Z between (17)–(20) and substituting the proper solution $W = W_0 \sin \pi z$, in the resultant equation, we obtain the dispersion relation

$$\begin{aligned} R_1 = {} & \left(\frac{1+x}{x}\right)(1+x+\mathrm{i}p_1\sigma_1) \\ & \cdot [\mathrm{i}\sigma_1 + \{1+F_1(1+x)\}(1+x)] \\ & + S_1 \frac{(1+x+\mathrm{i}p_1\sigma_1)}{(1+x+\mathrm{i}q\sigma_1)} \\ & + T_{A_1} \frac{(1+x+\mathrm{i}p_1\sigma_1)}{x[\mathrm{i}\sigma_1 + \{1+F_1(1+x)\}(1+x)]}, \end{aligned} \tag{23}$$

where $R_1 = \frac{g\alpha\beta d^4}{\upsilon\chi\pi^4}$, $S_1 = \frac{g\alpha'\beta' d^4}{\upsilon\chi'\pi^4}$, $T_{A_1} = \frac{4\Omega^2 d^4}{\nu^2\pi^4} = \left(\frac{2\Omega d^2}{\nu\pi^2}\right)^2$, $x = \frac{a^2}{\pi^2}$, $F_1 = \pi^2 F$, and $\frac{\sigma}{\pi^2} = \mathrm{i}\sigma_1$.

4. The Stationary Convection

When the instability sets in as stationary convection, marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (23) reduces to

$$\begin{aligned} R_1 = {} & \frac{(1+x)^3}{x}[1+F_1(1+x)] + S_1 \\ & + T_{A_1} \frac{1}{x[1+F_1(1+x)]}. \end{aligned} \tag{24}$$

To study the effect of stable solute gradient, rotation and couple-stress parameter, we examine the nature of $\frac{\mathrm{d}R_1}{\mathrm{d}S_1}$, $\frac{\mathrm{d}R_1}{\mathrm{d}T_{A_1}}$ and $\frac{\mathrm{d}R_1}{\mathrm{d}F_1}$ analytically.

Equation (24) yields

$$\frac{\mathrm{d}R_1}{\mathrm{d}S_1} = +1, \tag{25}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}T_{A_1}} = \frac{1}{x[1+F_1(1+x)]}, \tag{26}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}F_1} = \left(\frac{1+x}{x}\right)\left[(1+x)^3 - \frac{T_{A_1}}{[1+F_1(1+x)]^2}\right], \tag{27}$$

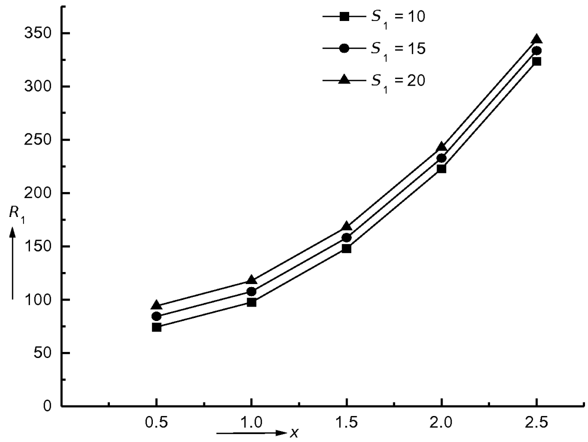

Fig. 1. Variation of R_1 with x for a fixed $F_1 = 5$, $T_{A_1} = 50$, for different values of $S_1 (= 10, 15, 20)$.

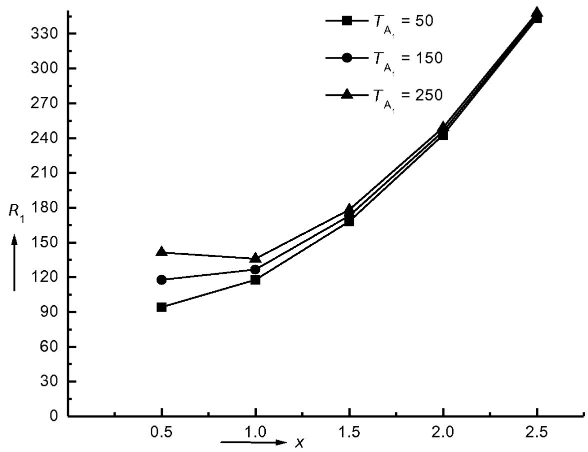

Fig. 2. Variation of R_1 with x for a fixed $F_1 = 5$, $S_1 = 25$, for different values of $T_{A_1} (= 50, 150, 250)$.

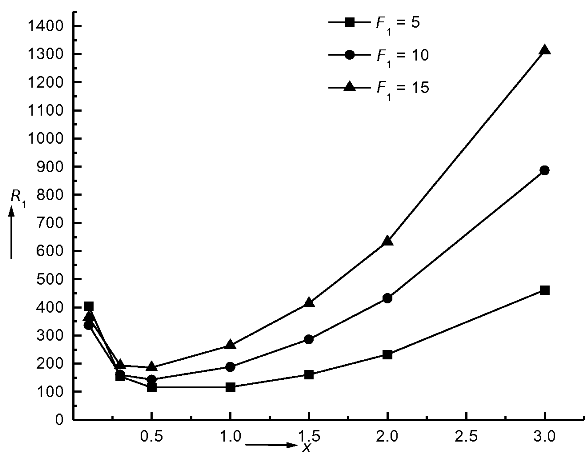

Fig. 3. Variation of R_1 with x for a fixed $S_1 = 10$, $T_{A_1} = 200$, for different values of $F_1 (= 5, 10, 15)$.

which imply that stable solute gradient and rotation have stabilizing effects on the system whereas couple-stress parameter has both stabilizing and destabilizing effects on the system in the presence of rotation.

Graphs have been plotted between R_1 and x for various values of S_1, T_{A_1} and F_1. It is also evident from Figures 1 – 3 that stable solute gradient and rotation have stabilizing effects and couple-stress parameter has both stabilizing and destabilizing effects on the system.

5. Stability of the System and Oscillatory Modes

Here we examine the possibility of oscillatory modes, if any, on the stability problem due to the presence of stable solute gradient and rotation. Multiplying (17) by W^*, the complex conjugate of W, integrating over the range of z and making use of (18) – (20) together with the boundary conditions (21), we obtain

$$\sigma I_1 + I_2 - \frac{g\alpha\chi a^2}{\nu\beta}[I_3 + p_1\sigma^* I_4] + \frac{g\alpha'\chi' a^2}{\nu\beta'}[I_5 + q\sigma^* I_6] + d^2[I_7 + \sigma^* I_8 + F I_9] + F I_{10} = 0, \tag{28}$$

where

$$\begin{aligned}
I_1 &= \int_0^1 (|DW|^2 + a^2|W|^2)\mathrm{d}z, \\
I_2 &= \int_0^1 (|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2)\mathrm{d}z, \\
I_3 &= \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)\mathrm{d}z, \quad I_4 = \int_0^1 |\Theta|^2\mathrm{d}z, \\
I_5 &= \int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2)\mathrm{d}z, \quad I_6 = \int_0^1 |\Gamma|^2\mathrm{d}z, \\
I_7 &= \int_0^1 (|DZ|^2 + a^2|Z|^2)\mathrm{d}z, \quad I_8 = \int_0^1 (|Z|^2)\mathrm{d}z, \\
I_9 &= \int_0^1 (|D^2Z|^2 + 2a^2|DZ|^2 + a^4|Z|^2)\mathrm{d}z, \\
I_{10} &= \int_0^1 (|D^3W|^2 + 3a^2|D^2W|^2 + 3a^4|DW|^2 + a^6|W|^2)\mathrm{d}z,
\end{aligned} \tag{29}$$

and σ^* is the complex conjugate of σ. The integrals $I_1 - I_{10}$ are all positive definite.

Putting $\sigma = \sigma_\mathrm{r} + \mathrm{i}\sigma_\mathrm{i}$ in (28) and equating real and imaginary parts, we have

$$\sigma_\mathrm{r}\left(I_1 - \frac{g\alpha\chi a^2}{\nu\beta}p_1 I_4 + \frac{g\alpha'\chi' a^2}{\nu\beta'}q I_6 + d^2 I_8\right) = -\left(I_2 - \frac{g\alpha\chi a^2}{\nu\beta}I_3 + \frac{g\alpha'\chi' a^2}{\nu\beta'}I_5 + d^2 I_7 + d^2 F I_9 + F I_{10}\right), \tag{30}$$

and

$$\sigma_\mathrm{i}\left(I_1 + \frac{g\alpha\chi a^2}{\nu\beta}p_1 I_4 - \frac{g\alpha'\chi' a^2}{\nu\beta'}q I_6 - d^2 I_8\right) = 0. \tag{31}$$

Equation (30) yields that σ_r may be positive or negative, i. e. there may be stability or instability in the presence of solute gradient and rotation in couple-stress fluid. It is clear from (31) that $\sigma_\mathrm{i} = 0$ or $\sigma_\mathrm{i} \neq 0$, which means that the modes may be non-oscillatory or oscillatory.

From (31) it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If $\sigma_\mathrm{i} \neq 0$, then (31) gives

$$I_1 = \frac{g\alpha'\chi' a^2}{\nu\beta'}q I_6 - \frac{g\alpha\chi a^2}{\nu\beta}p_1 I_4 + d^2 I_8. \tag{32}$$

Substituting in (30), we have

$$2\sigma_\mathrm{r} I_1 + I_2 + \frac{g\alpha'\chi' a^2}{\nu\beta'}I_5 + d^2 I_7 + d^2 F I_9 + F I_{10} = \frac{g\alpha\chi a^2}{\nu\beta}I_3. \tag{33}$$

Equation (33) on using Rayleigh-Ritz inequality gives:

$$\frac{(\pi^2+a^2)^3}{a^2}\int_0^1 |W|^2\mathrm{d}z + \frac{(\pi^2+a^2)}{a^2}\left\{F I_{10} + d^2 F I_9 + d^2 I_7 + \frac{g\alpha'\chi' a^2}{\nu\beta'}I_5 + 2\sigma_\mathrm{r} I_1\right\} \le \frac{g\alpha\chi}{\nu\beta}\int_0^1 |W|^2\mathrm{d}z. \tag{34}$$

Therefore, it follows from (34) that

$$\left[\frac{27\pi^4}{4} - \frac{g\alpha\chi}{\nu\beta}\right]\int_0^1 |W|^2\mathrm{d}z + \frac{(\pi^2+a^2)}{a^2}\left\{F I_{10} + d^2 F I_9 + d^2 I_7 + \frac{g\alpha'\chi' a^2}{\nu\beta'}I_5 + 2\sigma_\mathrm{r} I_1\right\} \le 0, \tag{35}$$

since the minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ with respect to a^2 is $\frac{27\pi^4}{4}$.

Now, let $\sigma_r \geq 0$, we necessarily have from (35)

$$\frac{g\alpha\chi}{\nu\beta} > \frac{27\pi^4}{4}. \tag{36}$$

Hence, if

$$\frac{g\alpha\chi}{\nu\beta} \leq \frac{27\pi^4}{4}, \tag{37}$$

then $\sigma_r < 0$. Therefore, the system is stable.

We summarize, under condition (37), the system is stable and under condition (36) the system becomes unstable.

In the absence of stable solute gradient and rotation, (31) reduces to

$$\sigma_i \left(I_1 + \frac{g\alpha\chi a^2}{\nu\beta} p_1 I_4 \right) = 0, \tag{38}$$

and the terms in brackets are positive definite. Thus, $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is valid for the couple-stress fluid in the absence of stable solute gradient and rotation. The presence of each, the stable solute gradient and the rotation brings oscillatory modes (as σ_i may not be zero) which were non-existent in their absence.

6. The Case of Overstability

Here we discuss the possibility of instability or overstability. Since we wish to determine critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (23) will admit a solution with σ_1 real.

Equating the real and imaginary parts of (23) and eliminating R_1 between them and setting $c_1 = \sigma_1^2$, $b = 1 + x$, we obtain

$$A_2 c_1^2 + A_1 c_1 + A_0 = 0, \tag{39}$$

where

$$\begin{aligned} A_2 &= [1 + p_1(1 + F_1 b)] q^2 b^2, \\ A_1 &= S_1(b-1)b(p_1 - q) + T_{A_1} q^2 b(p_1 - 1) \\ &\quad + b^4\{1 + q^2(1 + F_1 b)^2\}\{1 + p_1(1 + F_1 b)\} \\ &\quad + T_{A_1} p_1 q^2 F_1 b^2, \\ A_0 &= S_1(b-1)(1 + F_1 b)^2 b^3 (p_1 - q) + T_{A_1} b^3 (p_1 - 1) \\ &\quad + b^6 (1 + F_1 b)^2 \{1 + p_1(1 + F_1 b)\} + T_{A_1} p_1 b^4 F_1. \end{aligned} \tag{40}$$

Since σ_1 is real for overstability, both the values of c_1 $(= \sigma_1^2)$ are positive. Equation (39) is quadratic in c_1 and does not involve any of its roots to be positive,if

$$p_1 > q \text{ and } p_1 > 1, \tag{41}$$

which imply that

$$\chi < \chi' \text{ and } \chi < \nu. \tag{42}$$

Thus $\chi < \chi'$ and $\chi < \nu$ are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

7. Conclusions

The effect of uniform vertical rotation on thermosolutal convection in a layer of couple-stress fluid heated and soluted from below is considered in the present paper. The investigation of thermosolutal convection is motivated by its interesting complexities as a double diffusion phenomena as well as its direct relevance to geophysics and astrophysics. The main conclusions from the analysis of this paper are as follows:

(i) For the case of stationary convection, the stable solute gradient and rotation have stabilizing effects on the system, whereas the couple-stress parameter has both stabilizing and destabilizing effects.

(ii) It is also observed from the Figures 1-3 that stable solute gradient and rotation have stabilizing effects whereas couple-stress parameter has both stabilizing and destabilizing effects on the system.

(iii) It is observed that the presence of each, the stable solute gradient and the rotation, brings oscillatory modes in the system, which were non-existent in their absence.

(iv) It is found that if $\frac{g\alpha\chi}{\nu\beta} \leq \frac{27\pi^4}{4}$, the system is stable and under the condition $\frac{g\alpha\chi}{\nu\beta} > \frac{27\pi^4}{4}$, the system becomes unstable.

(v) It is observed that in the absence of stable solute gradient and rotation, oscillatory modes are not allowed and the principle of exchange of stabilities is valid.

(vi) The conditions $\chi < \chi'$ are $\chi < \nu$ are sufficient for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

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